**R N S INSTITUTE OF TECHNOLOGY**

**DEPARTMENTOF MATHEMATICS**

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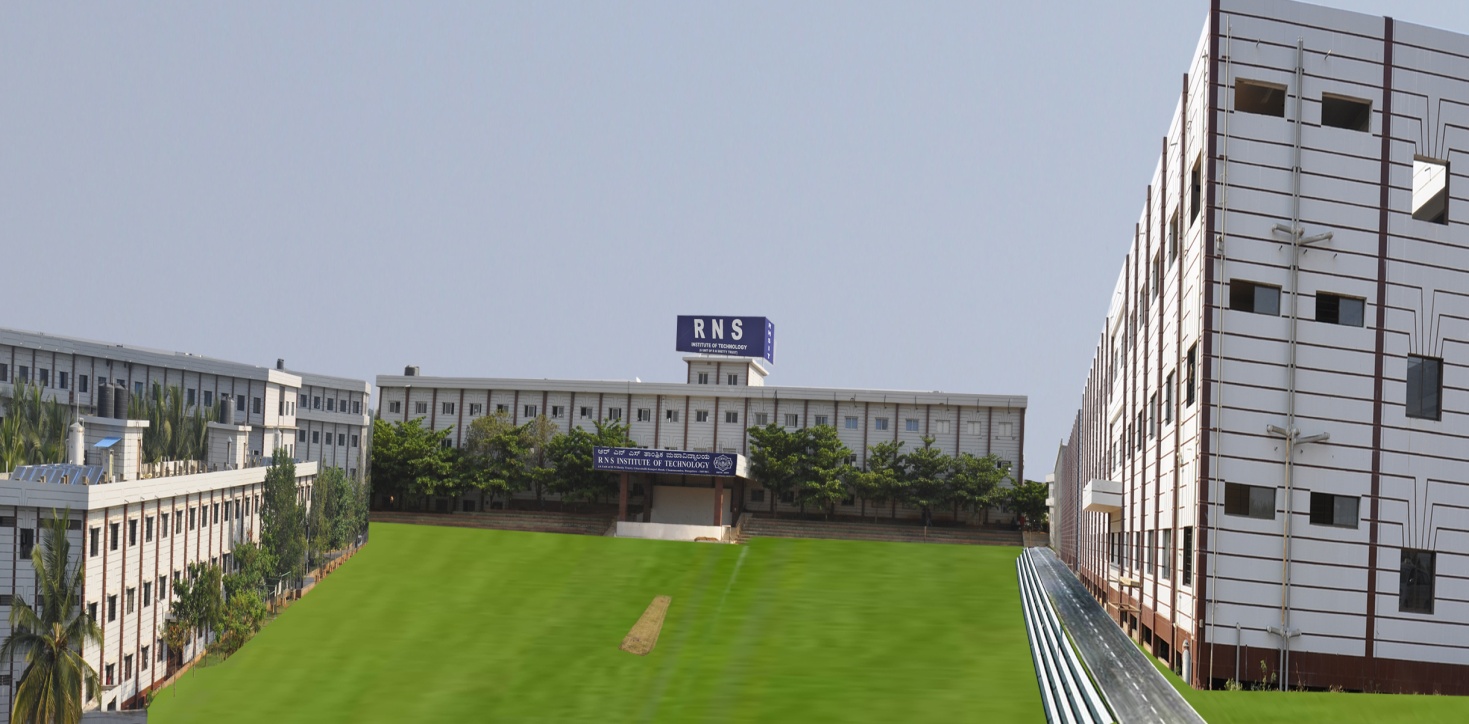
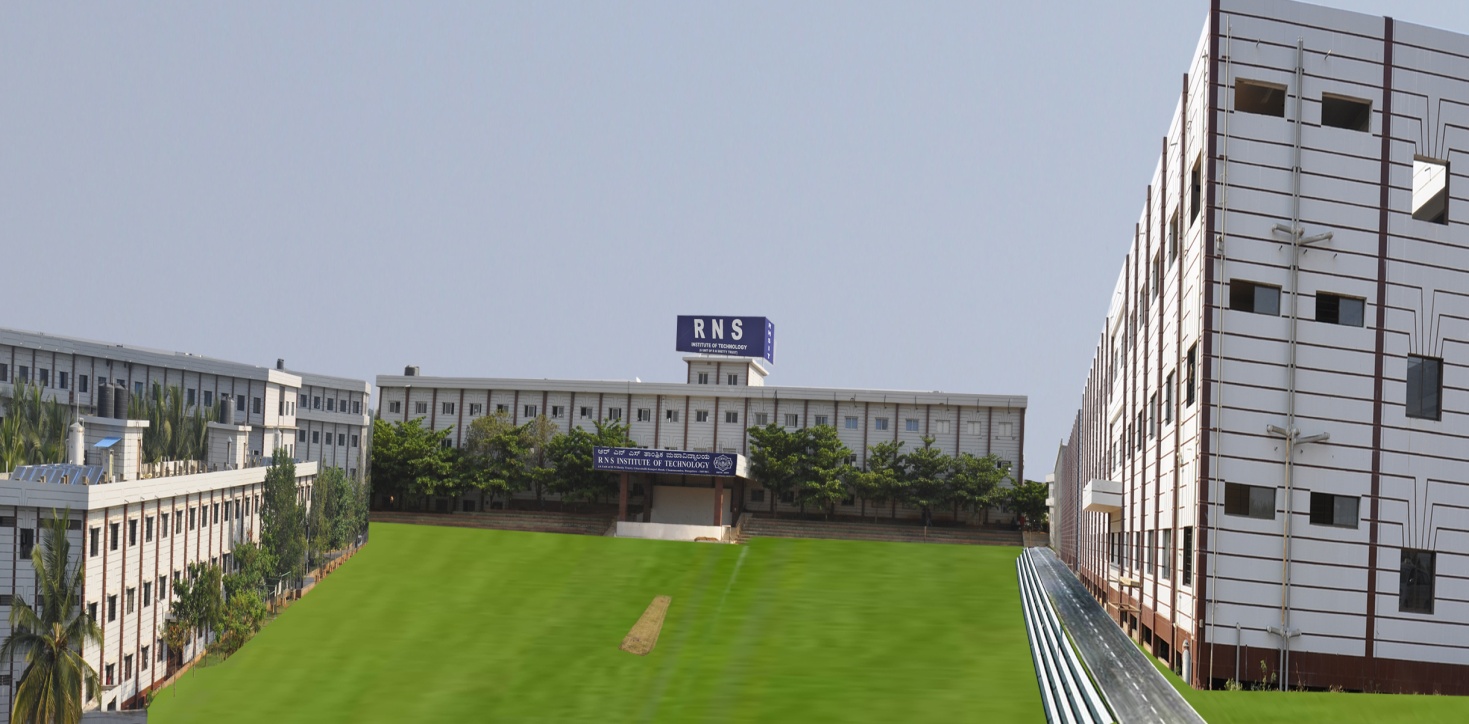
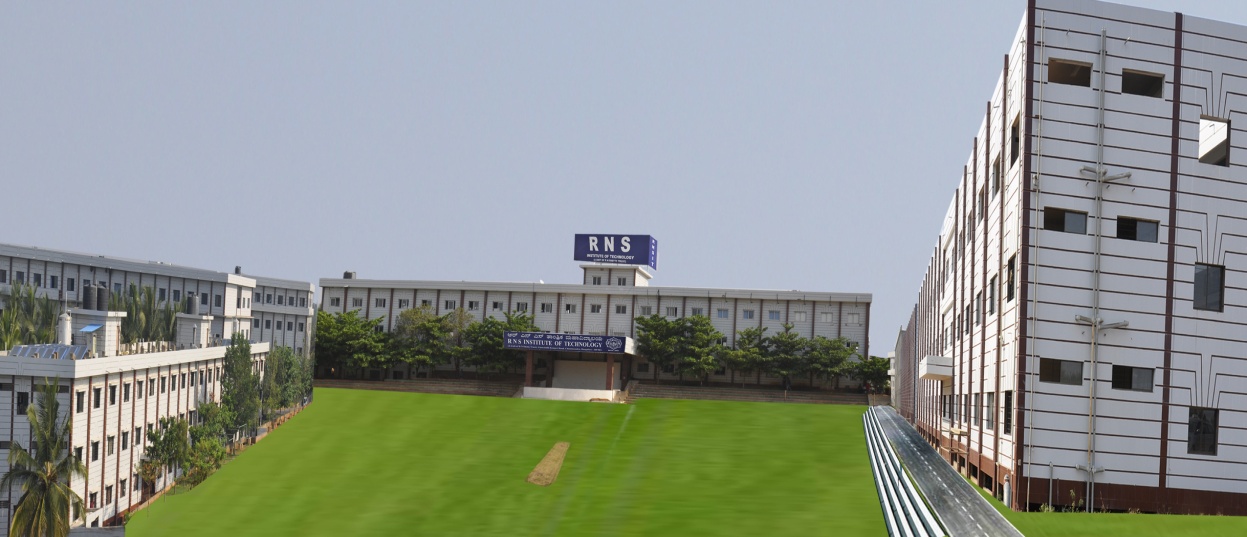
**STUDY MATERIALS**

**FOR**

**MATHEMATICS**

**VTU NEW SYLLABUS**

**MODULE-1**

****

**Differential calculus-1**

**MODULE-1**

* **Trigonometric Relation**

**1.** 

**2.** 

**3.** 

**4.** 

**5.**  **6.** 

**7.** 

**8.**  **9.** 

**10.**  **11.** 

* **Logarithmic Relation**

**1.**  **2.**  **3.**  **4.** 

* **Leibnitz Theorem**



* **Polar Curves**

**1.**  **2.** **3.**

**4.** **5.** 

**6.** **7.** 

**8.**  **9.** 

**10.**  **We have**  **11.** 

**12.**  **13.**  **14.**  **15.** 

**16.** 

* **nth Derivatives of some standard functions:**

|  |  |  |
| --- | --- | --- |
| **SL. NO.** | **Functions (y)** | **nth derivative (yn)** |
| **1.** |  |  |
| **2.** |  |  |
| **3.** |  |  |
| **4.** |  |  |
| **5.** |  |  |
| **6.** |  |  |
| **7.** |  |  |
| **8.** |  |  |
| **9.** |  |  |
| **10.** |  |  |
| **11.** |  |  |
| **12.** |  |  |
| **13.** |  |  |
| **14.** |  |  |
| **15.** |  |  |

**CHAPTER – 1 : Nth Order Derivatives**

Differential Calculus is one of the most revered fields of study. Measure the rate of change of a given function. Differential Calculus is used and applied in all the Engineering departments. Higher order derivatives give the characteristics of a function and this function can be any practical relation of a moving body, circuit mechanism, constructional analysis, and many more. It tells the maximum and minimum variations and slope characteristics.

* The first derivative,  denotes velocity at a given time
* The second derivative,  denotes acceleration at a time
* The third derivative,  denotes the [jerk or jolt](http://en.wikipedia.org/wiki/Jerk_%28physics%29) at time , an important quantity in engineering and motion control
* The fourth derivative denotes the [jounce](http://en.wikipedia.org/wiki/Jounce) at time; the jounce is also used in studying motion, and in studying the cosmological equation of state.

Fifth and sixth derivatives of position are also important in some applications/theoretical physics studies, but they have no universally accepted name.

Fifth derivative and curve fitting are used to do DNA analysis and population matching.

And higher derivatives are also used for approximating functions using Taylor polynomials, which can be useful when a certain amount of precision is required.

Nth order is used to formulate a generalised function for any order derivative. This makes it easier for calculation

Leibnitz was a German [polymath](https://en.wikipedia.org/wiki/Polymath) and philosopher who occupies a prominent place in the [history of mathematics](https://en.wikipedia.org/wiki/History_of_mathematics) and the [history of philosophy](https://en.wikipedia.org/wiki/History_of_philosophy), having developed [differential and integral calculus](https://en.wikipedia.org/wiki/Differential_and_integral_calculus) [independently](https://en.wikipedia.org/wiki/Multiple_discovery) of [Isaac Newton](https://en.wikipedia.org/wiki/Isaac_Newton). [Leibniz's notation](https://en.wikipedia.org/wiki/Leibniz%27s_notation) has been widely used ever since it was published. It was only in the 20th century that his [Law of Continuity](https://en.wikipedia.org/wiki/Law_of_Continuity) and [Transcendental Law of Homogeneity](https://en.wikipedia.org/wiki/Transcendental_Law_of_Homogeneity) found mathematical implementation (by means of [non-standard analysis](https://en.wikipedia.org/wiki/Non-standard_analysis)). He became one of the most prolific inventors in the field of [mechanical calculators](https://en.wikipedia.org/wiki/Mechanical_calculator). While working on adding automatic multiplication and division to [Pascal's calculator](https://en.wikipedia.org/wiki/Pascal%27s_calculator), he was the first to describe a [pinwheel calculator](https://en.wikipedia.org/wiki/Pinwheel_calculator) in 1685 and invented the [Leibniz wheel](https://en.wikipedia.org/wiki/Leibniz_wheel), used in the [arithmometer](https://en.wikipedia.org/wiki/Arithmometer), the first mass-produced mechanical calculator. He also refined the [binary number](https://en.wikipedia.org/wiki/Binary_number) system, which is the foundation of virtually all digital computers.



**Module-1 Syllabus**

Determination of nth order derivatives of Standard functions - Problems. Leibnitz’s theorem (without proof) - problems. **Polar Curves** - Angle between the radius vector and tangent, Angle between two curves, Pedal equation of polar curves. Derivative of arc length - Cartesian, Parametric and Polar forms (without proof) - problems. Curvature and Radius of Curvature – Cartesian, Parametric, Polar and Pedal forms (without proof) -problems

* **Find the nth derivative of .**

Solution: Let

Differentiating w r to x

.

**Case(i):** If m is a positive integer and m > n. Then, we have,

In particular,

**Case(ii):** If (a positive integer). Then

In particular,

**Case(iii):** If m is a positive integer and .

Then,

**Case(iv):** If m is a positive integer and .

Then

In particular,

**Case(v):** If .

Then

In particular,

* **Find the nth derivative of .**

Solution: Let

Differentiating w r to x

; ; ;

In particular,

* **Find the nth derivative of .**

Solution: Let

Differentiating w r to x

In particular,

* **Find the nth derivative of .**

Solution: Let

Differentiating w r to x

In particular,

* **Find the nth derivative of .**

Solution: Let

Differentiating w r to x

Setting and

so that ,

Proceeding like this, we get

In particular,

* **Find the nth derivative of .**

Solution: Let

Differentiating w r to x

Setting and

so that ,

Proceeding like this, we get

In particular,

**Type-1**

**1. Find the nth derivative of the following functions:**

(*i*)  **(VTU., 2006) (*ii*) (*iii*)**

**(*iv*) (VTU., 2009) (v)**

* (i) Let

Therefore,

* (ii) Let

Therefore,

* (iii) Let

Therefore,

* (iv) Let

* (v) Let

Therefore,

**Type-2**

**In this type, before applying derivatives of standard functions, we convert improper fractions into a sum of a polynomial and a proper functions and use partial fractions.**

**2. Find the nth derivative of the following functions:**

(i) (VTU 2009)

Solution: Let (1)

Consider,

(2)

putting in (2), we get

and putting in (2) we get .

substituting the values of *A* and *B* in (1),

* (ii) (VTU., 2005,2013)

Solution: Since the given fraction is an improper fraction.

To convert it into proper fraction divide the numerator polynomial from the denominator polynomial.

---------------------

Let (1)

Consider, (2)

. (3)

putting in (3), we get

and putting in (3), we get .

Substituting the values of *A* and *B* in (2),

and the equation (1) gives,

(iii)

Solution: Let (1)

Consider,

. (2)

putting in (2), we get

and putting in (2), we get .

Substituting the values of *A* and *B* in (1),

* (iv)

Solution: Let (1)

Consider,

. (2)

putting in (2), we get

and putting in (2), we get .

Substituting the values of *A* and *B* in (1),

* (v)

Solution: Let (1)

Consider

(2)

putting in (2), we get

and putting in (2), we get

collect the coefficient of on both sides of equation (2),

we get

Substituting the values of *A,B* and *C* in (1),

* (v) **VTU** **(2017)**

Solution: Let (1)

Consider

(2)

putting in (2), we get

and putting in (2), we get

collect the coefficient of on both sides of equation (2),

we get

Substituting the values of *A,B* and *C* in (1),

* (vi)

Solution: Let (1)

Consider

. (2)

Taking in (2), we get

and taking in (2), we get .

Equation (1) gives

and

Put so that and

Then, and

and

* (vii)

Solution: Let (1)

. (2)

taking , in (2), we get

and taking in (2), we get .

Put so that and

Then, and

and

* (viii)

Solution: Let (1)

Consider

(2)

By putting in (2), we get ,

Collecting the coefficient of on both sides we get so that ,

Collecting the coefficient of on both sides we get .

Subzstituting *A, B and C* in (1), we get

[using the above problem]

**Type-3**

**In this type we discuss to find nth derivatives of functions whose derivatives can be obtained using the results of previous type.**

**3. Find the nth derivative of (i) (ii) (iii) (iv)**

* (i) Let

* (ii) Let

* (iii) Let

* (iv) Let

**Type-4**

**In this type we discuss to find nth derivatives of logarithmic functions.**

**4. Find the nth derivative of the following:**

(i) (VTU 2010) (ii) (iii)

(iv) (v) .

* (i) Let

* (ii)

* (iii)

* (iv)

* (v)

**Type-5**

**In this type we discuss to find nth derivatives of product of exponential and sine and exponential and cosine functions.**

**5. Find the nth derivative of the following functions:**

**(i) (VTU 2010S) (ii) (iii)**

**(iv) (v)**

* **Solution: (i)**

* **Solution: (ii)**

* **(iii)**

Solution:

* **(iv)**

Solution:

* **(v)**

Solution:

Exercise:

**1. Ans:**

**2. Ans:**

**3. Ans:**

**4. Ans:**

**5. Ans:**

**6. Ans:**

**Problems: Find the nth derivative of the following:**

**1. Ans:**

**2. Ans:**

**Problem: Find the nth derivative of the following:**

**1. Ans:**

**2. Ans:**

**3. Ans:**

**4. Ans:**

**5. Ans:**

**6. Ans:**

**7. Ans:**

**8.**

**Ans:**

**9.**

**Ans:**

**10.**

**Ans:**

**Problems:**

**Find the nth derivative of the following:**

**1. Ans:**

**2. Ans:**

**3. Ans:**

**4. Ans:**

**5. Ans:**

**6. Ans:**

**7. Ans:**

**8. Ans:**

**9. Ans:**

**10. Ans:**

**11. Ans:**

**Find the nth derivative of the following:**

**1.**

**Ans:**

**2.**

**Ans:**

**3. Ans:**

**4. Ans:**

**5.**

**Ans:**

**6.**

**Ans:**

**7.**

**Ans:**

**8.**

**Ans:**

**9.**

**Ans: (2017)**

**Type-6**

**In this type we discuss to find nth derivatives of given functions use the Leibntiz’s theorem.**

**LEIBNITZ THEOREM**

The invention of calculus is now credited jointly to Isaac Newton (1642 - 1727) and Gottfried W. Leibniz (146 - 1716) the following theorem given by Leibniz helps us to find the nth derivative of the product of two functions.

Statement : If and be two functions of possessing derivatives of the nth order then

(1)

(where suffixes denote the differentiation of those orders and and

Working rule

1. Choose properly and , since will become zero for some so that the process   
 terminates at some stage.

2. In all the standard type of problems use your discretion for getting the result required.

**Problems on Leibnitz’s theorem:**

1. Find the nth derivatives of the following functions

(i) (ii) (iii) (iv)

Solution:

(i) To find

Taking and , applying Leibnitz theorem, we have

(ii) To find

Solution: Taking and applying Leibnitz theorem we have

(iii) To find

Solution: Let then

Multiplying both sides by , we have

or (1)

Differentiating both sides of equation (1) times using Leibnitz theorem, we have

(iv) To find

Solution: By Leibnitz theorem, we have

2. Find the nth derivative of .

Solution: Let

Let

etc

Note: Take always is algebraic i.e.,

By Leibnitz’s theorem, we get

3. Find the nth derivative of

Solution: Let,

etc.

By Leibnitz’s theorem, we get

4. Find the nth derivative of

Solution: Let

etc.

By Leibnitz’s theorem, we get

5. Find the nth derivative of

Solution: Let

etc.

By Leibntiz’s theorem,

6. Find the nth derivative of

Solution: Let

etc.

7. Evaluate

Solution: Consider

etc

By Leibnitz’s theorem, we have

8. If show that . **(VTU 2006)**

Solution: Let or (1)

Differentiate (1) n times by using Leibnitz’s theorem taking as the first function, we have

9. If show that .

Solution: Let

Differentiate w r t x,

Differentiate every term times by using Leibnitz’s theorem

or

10. If prove that and hence show that

(VTU 2001)

Solution: Let

(1)

Putting the values for n = 1, 2, 3, 4, 5, … in (1), we get

n = 1,

n = 2,

n = 3,

and so on.

Similarly,

**Type-7**

1. If then show that

(i)

(ii) (VTU 2003)

Solution: Let

Differentiate with respect to x

Differentiate with respect to x once again

from given equation

(1)

Differentiate every terms of equation(1) times by using Leibnitz’s theorem, we get

I term : .

II term :

etc.

III term : substituting these values in Leibnitz theorem, we get

**2.** If or or prove that **(2017)**

**(VTU July 2004)**

Solution: Let

Differentiate with respect to x.

Differentiate every term times by using Leibnitz’s theorem,

i. e.,

3. If or or If , show that

(i)

(ii) . (VTU : July 2004)

Solution: (i) Consider

i.e.,

Eliminating we get

Differentiate with respect to x,

Squaring on both sides,

Differentiate every term times by using Leibnitz’s theorem,

4. If show that (i) ,

(ii) (VTU 2005)

Solution: (i) Consider

i.e.,

Eliminating we get

Differentiate with respect to x,

Squaring on both sides,

Differentiate every term times by using Leibnitz’s theorem,

5. If show that

or If . Show that satisfies the equation   
 (VTU Feb2005)

Solution: Let

Differentiate with respect to x again

Differentiate every term times by using Leibnitz theorem,

6. If show that (i) ,

(ii) (VTU July2006, 2007)

Solution: (i) Let

Differentiate with respect to x,

Squaring on both sides

Differentiate with respect to x

(ii) Differentiate every term times by using Leibnitz’s theorem.

7. If prove that . Also S T (i)

(ii) **(VTU 2008S, 2011, 2013, 2017)**

Solution: Given

[multiplying throughout by ]

Taking power on both sides we get

Differentiating with respect to x

Squaring on both sides

or

(ii) Differentiate every term times by Leibnitz’s theorem.

Similarly we get for

8. If show that (i)

(ii) Hence find (VTU 2009)

Solution: (i) Let

Differentiate with respect to x,

Squaring on both sides

Differentiate with respect to x

(ii) Differentiate every term times by using Leibnitz’s theorem.

when *x = 0;*

Also replace n by n-2, we get and

9. If , prove that . Hence show that   
 (VTU 2009)

Solution: Given

Differentiate with respect to x,

Squaring on both sides

Differentiate again with respect to x,

Differentiate every term times by using Leibnitz’s theorem.

(1)

put x = 0 in (1),

(2)

This is a recurrence relation in n

Taking n = 0, 1, 2, ... ... in (2) we get

n = 0 in (2),

n = 1 in (2),

n = 2 in (2),

n = 3 in (2),

n = 4 in (2),

n = 5 in (2),

n = 6 in (2),

n = 7 in (2), and so on.

Hence

10. If show that (i) ,

(ii) at x = 0 (VTU 2009)

Solution: (i) Let

Differentiate with respect to x,

Squaring on both sides

Differentiate with respect to x

or

(ii) Differentiate every term times by using Leibnitz’s theorem.

when *x = 0;*is the required result.

11. If show that (VTU 2010, 2014)

Solution: Let

Squaring on both sides, we get

Differentiate with respect to x,

Differentiate again with respect to x,

(ii) Differentiate every term times by using Leibnitz’s theorem.

12. If show that

(VTU 2010S)

Solution: Let

Differentiate with respect to x,

Squaring on both sides, we get

Differentiate again with respect to x,

Differentiate every term times by using Leibnitz’s theorem.

13. If show that (i)

(ii) . (VTU 2012)

Solution: (i)

Differentiate with respect to x,

Squaring on both sides

Differentiate once again with respect to x,

(ii)Differentiate every term times by using Leibnitz’s thm, we get,

14. If show that .

Hence determine the value of when (VTU 2013)

Solution: (i)

Differentiate with respect to x,

Squaring on both sides

Differentiate again with respect to x,

(ii)Differentiate every term times by using Leibnitz’s theorem, we get,

(1)

put x = 0 in (1), we get

(2)

when

put in (2)

n = 1,

n = 2,

n = 3,

when n is odd

when n is even.

15. If show that

(i) ,

(ii) (VTU 2004, 2014)

16. If show that .

Hence determine the value of all derivatives of with respect to x when

Solution: Let (1)

Differentiate with respect to

(2)

Differentiate times by using Leibnitz’s theorem.

(3)

Putting in (1), (2) and (3)

and (4)

Putting in (4)

n = 1 in (4),

n = 2 in (4),

n = 3 in (4),

n = 1 in (4),

Hence when is odd

When is even .

17. If or or show that

(i) (ii)

Solution: (i)

Squaring on both sides

Differentiate with respect to x

(ii) Differentiate every term times by using Leibnitz’s theorem we get,

18. If or or show that (2017)

(i) (ii) .

Solution: Let

Differentiate with respect to x

Differentiate with respect to x

(ii) Differentiate every term times by using Leibnitz’s theorem.

19. If show that

(i) (ii)

Solution: (i) Consider

Differentiate with respect to x,

Once again differentiate with respect to x,

(ii) Differentiate every term times by using Leibnitz’s theorem,

20. If show that . Hence deduce that

Solution: Let

Differentiate with respect to x,

Differentiate with respect to x,

Differentiate times by Leibnitz theorem, we get

Example 21. If show that

Solution: Consider the given

i. e.,

Taking power both sides

Squaring on both sides

Diff. w. r. t.

Diff. every term times by using Leibnitz theorem,

I term :

.

II term :

...etc

III term : Substituting all these in Leibnitz theorem.

**Polar Curves**

Polar Curves comprises of Curvature, Radius of Curvature, Arc length and so on.

Curvature is a measure of rate of change of a bentness. Straight line has zero bentness while a circle has constant bentness.

These help was to calculate the orbits of satellites, gravitational theorems, even metallurgical analysis is dependent on these, in automobile design, aerodynamics, and in fluid dynamics.

Polar curves have many applications in engineering fields. Some application of differential calculus to geometry will discuss in this section. Other than Cartesian system of coordinates, there is another system to represent a point and curve analytically in a plane known as the polar coordinate system.

Let us choose a fixed point on the plane and call it as the pole. A fixed line drawn form the pole is called initial line or polar axis. Then the position of any point P in the plane, is obtaining by joining the points O and P. If the distance OP = r is called radius vector and X, the vectorial angle. The angle is considered positive when it is measured in the anti-clockwise direction and clearly r and Then (are called the polar coordinates of the point P.

Let be the Cartesian coordinates of the point P. Then we have

P(r,θ)

y

x

Y

X

o

(1)

gives (2)

The relation (1) finds Cartesian in terms of polar coordinates. Conversely relation (2) gives polar coordinates in terms of Cartesian coordinates. The equation of a curve in polar coordinates will be in the form A curve specified by a polar equations is referred to as a polar curve.

**Angle between Radius Vector and Tangent**

1. With usual notation prove that

Y

o

P(r,θ)

X

θ

ϕθ

where is the angle between radius vector and tangent.

Let be any point on a polar curve

(1)

and (2)

Draw a tangent to the curve at , meets the at the point . Let the angle between radius vector and the tangent be and be the angle between tangent and positive .

Let be the Cartesian coordinates of the point then we have

,

(3)

and slope of tangent

fig

(4)

[from (3)]

Dividing numerator and denominator by we get

(5)

from (4) and (5), we get, .

Thus .

**Angle of intersection of two polar curves**

The angle of intersection of two curves is the angle between their tangent at that point. Let the curves

and intersect at . Let and be the angles between the common radius vector OP and tangents PT1 and PT2. Hence the angle between the two tangents is equal to . Therefore the acute angle of intersection of the curves is equal to .

where and are determined by the formula.

for

for

Suppose the angle between two curves is given by

or equivalently , then we say that the curves intersect orthogonally.

**Problems:**

**Find the angle of intersection between the following curves:**

**1. and VTU DEC 2011**

**Solution: Consider the given curves**

**(1) and (2)**

**Taking log for the both curves**

**Differentiate with respect to Differentiate with respect to**

**and**

**2. and VTU 2012 D**

**Solution: Consider the given curves (1) and (2)**

**Solving the equations**

**Differentiate (1) with respect to**

**and**

**Differentiate (1) with respect to**

**At**

**3. and VTU 2008S**

**Solution: Consider the given curves (1)**

**and (2)**

**Taking log for the both curves**

**Differentiate with respect to Differentiate with respect to**

**therefore the angle between the two curves is given by**

**(3)**

**Solving the equations (1) and (2)**

**and**

**therefore**

**4. and VTU 2004**

**Solution: Consider the given curves (1) and (2)**

**Taking log for the both curves**

**Differentiate with respect to Differentiate with respect to**

**[dividing Nr and Dr by ]**

**5. and (VTU F2003, 2006)**

**Solution: Consider the given curves (1) and (2)**

**solving the equations (1) and (2),**

**Differentiate with respect to Differentiate with respect to**

**; For all**

**when ,**

**6. Show that and cut orthogonally. (VTU 2003, F06,11S)**

Solution: The curves are

For the curve (1) we have

For the curve (2) we have

But

the curves cut orthogonally.

**7. Show that and cut orthogonally.**

Solution : Consider the given curve

Taking log on both sides of given curves, we have

and

Differentiate with respect to we get

So that the curves cut

**9. Show that the tangent to the Cardioid at the points and   
 are respectively parallel and perpendicular to the initial line.**

**Solution: Given the curve**

Differentiate with respect to we get

now we have

when

This shows that the tangent and the initial line co insides.

when

This shows that the tangent and the initial line perpendicular.

**10. Find the angle of intersection of curves .**

Solution: Let the curves

Taking log for both curves

Differentiate with respect to

.

But, and

Solving given equations, we get

At

Let be the angle of intersection of two curves

**11. Show that the curves , intersect each other orthogonally.**  Solution: Consider the given curve (1)

(2)

Differentiate (1) with respect to , we get

Consider

Differentiate with respect to , we get

So the curves intersect orthogonally.

**12. Show that the curves and cut orthogonally**.

Solution: The given curves are

(1)

(2)

Differentiate (1) with respect to we get

For the curve (2),

So the curves cut orthogonally.

**13. Show that the curves and intersect each other orthogonally.**

Solution: The given curves are (1) and (2)

For the curve (1) we have

For the curve (2) we have

Eliminating r between equations (1) and (2), we get

i.e.,

When

When

The two points of intersection are and

At

and at

Hence curves intersect orthogonally.

**14. Show that the curves intersect each other orthogonally**.

Solution: The given curves are (1) (2)

Taking log both sides for the curves

and

Differentiate with respect to , we get

But

So the curves intersect orthogonally.

**15. Show that and cut orthogonally. (VTU 2017)**

Solution: The curves are

For the curve (1) we have

For the curve (2) we have

But

the curves cut orthogonally.

**16. Show that and cut orthogonally. (2017)**

Solution: The curves are

For the curve (1) we have

For the curve (2) we have

But

the curves cut orthogonally.

**Questions Bank 1.3**

**1. Find the angle between radius victor and the tangent for each of the following curves.**

(a) Ans:

(b) Ans :

(c) . Ans :

(d) Ans :

(e) Ans :

(f)

(g)

**2. Prove that the normal at any pint on the curve with   
 the initial line.**

**3. Find the slope of the following curves.**

(a)

(b)

(c)

(d)

**5. Find the angle of intersection of each of the following pairs of curves.**

(a)

(b)

(c)

(d)

(e)

(f) and

**6. Show that each of the following pairs of curves intersect orthogonally.**

(a) and

(b) and

(c) and

(d)

(e) and

(f)

**Length of the perpendicular form pole to the Tangent**

Consider a point on the curve

Draw perpendicular form the pole to the tangent at P.

Form Fig.

p

0

X

M

r

and

Then we have

or squaring and inverting, we get

(1)

Note: This formula can be expressed in another form as

Let then,

substituting these in (1), we get

**1.4 Pedal Equation**

Any equation containing p and r (without ) is called pedal equation or p-r equation and where p is length of perpendicular from the pole to the tangent and r the radius vector.

Note: (i) If (1) (2) and (3)

Eliminating and from these equations we get pedal equation.

(ii)

If is express in terms of explicitly then we use (i) otherwise we use (ii).

**Example: 1. Find the pedal equation of the curve**

Solution: Consider the given curve

Differentiate with respect to , we get

We have,

Squaring on both sides;

**Example: 2 find the pedal equation of the curve (VTU 2009)**

Solution: Consider the given curve

Differentiate with respect to , we get

We have,

Squaring on both sides;

**Example: 3. find the pedal equation of the curve**

Solution: Taking log on both sides

Differentiate with respect to , we get

But

We have

(1)

Consider the given equation

It can be written as

or

Thus is the required pedal equation.

**Example: 4. Find the pedal equation of the curve**

Solution: The given curve is (1)

Taking log on both sides for the given curve,

Differentiate with respect to ,

We have

(2)

Substitute (2) in (1), we get

is the required pedal equation.

**Example: 5. Find pedal equation of the curve**

Solution: The given curve

Differentiate with respect to we get

Since we are not able to eliminate in ,

The equation is obtain by

i.e.,

[ from (1) ]

i.e.,

Hence is the required pedal equation.

**Example:6. Find the pedal equation of the curve**

Solution: The given equation is (1)

Taking log on both sides

Differentiate with respect to , we get

We have

or

Substituting this in (1), we get

Hence, is the required pedal equation.

**Example : 7. Find the pedal equation of the curve**  **(VTU 2010)**

Solution: The equation of the given curve (1)

Taking log on both sides

Differentiate with respect to , we get

We have

is the required pedal equation.

**Example: 8. Find the pedal equation of the curve,**  **(VTU F2005)**

Solution: consider the equation

Taking log on both sides,

Differentiate with respect to

Consider

squaring and taking reciprocal we get

or

is the required pedal equation.

**Example 9. Find the pedal equation of the curve . (VTU 2010, 2014J)**

Solution: The equation of the curve

Taking log on both sides,

Differentiate with respect to

Consider

is the required pedal equation.

10. **Find the Pedal equation of the curve (VTU 2004)**

Solution: The equation of the curve

Taking log on both sides,

Differentiate with respect to

Consider

is the required pedal equation.

**11. Find the Pedal equation of curve (VTU 2010)**

Solution: The equation of the curve

Taking log on both sides,

Differentiate with respect to

Consider

Squaring on both sides

is the required pedal equation.

**12. Find the Pedal equation of curve (VTU 2007)**

Solution: The equation of the curve

Taking log on both sides,

Differentiate with respect to

Consider

Squaring and taking reciprocal we get

or

is the required pedal equation.

**13. find the pedal equation of the curve**

Solution: Taking log on both sides

Differentiate with respect to , we get

But

We have

(1)

Consider the given equation

It can be written as

or

Thus is the required pedal equation.

**1.Find the pedal equation for the following curves**

(a)

(b)

(c)

(d)

(e)

(f) :

(h)

(i)

(j)

**2. Show that the pedal equation of the curve is**

**3. Show that the length of the perpendicular from the pole to the tangent at the point on the curve is equal to .**

**Derivative of arc length:**

* **If the equation of the curve is we have**
* **If the equation of the curve is then**
* **If the equation of the curve is then**
* **For the curve we have**
* **For the curve we have**

**Problems:**

**1. Calculate for the following curves:**

(i) (ii)

**2. Find for the curve (VTU 2007)**

**3. Find for the following curves:**

(i) (VTU 2004) (ii)

(ii) (VTU 2007)

**4. For the curves prove that (VTU 2005)**

**5. With the usual meanings for for the polar curve show that**

**(VTU 2000)**

**Curvature:**

Let P be any point on a given curve and Q a neighbouring point.

Y

o

P

X

θ

Q

Let arc AP = s and arc . Let the tangents at P and Q

make angle and with the x-axis, so that the angle

between the tangents at P and Q is . In moving from the

point P to Q through a distance , the tangents has turned through the angle . This is called the total bending or total curvature of the arc PQ.

**Therefore the average curvature of arc**

The limiting value of average curvature when Q approaches P (i,e., ) is defined as the curvature of the curve at P. Thus curvature

**The unit of measurement of a curvature is radians per unit length.**

**Radius of curvature: The reciprocal of the curvature of a curve at any point p is called the radius of curvature at p and is denoted by and defined by .**

**1. Radius of curvature for Cartesian curve is given by:**

We know that or

Differentiating both sides with respect to x,

**Note: 1. The radius of curvature is positive or negative according as is positive or negative(i.e according as the curve is concave upwards or downwards).**

**2. Since the radius of curvature is independent of the choice of coordinate axes, therefore interchanging x and y we get**   
 this formula is useful when the tangent is perpendicular to the x-axis. i.e or .

**2. Radius of curvature for parametric equation is given by:**

The equation of the curve in parametric form is , then

**where dashes represent the differentiations with respect to t**

**Substituting the values of and in**  we get

or

**Problems: (Cartesian form)**

**1. Find the radius of curvature at the point of the Folium   
 (VTU2008)**

Solution: Let the given curve is **(VTU 2017)**

Differentiating with respect to x, we get

(1)

Differentiating (1),

At the radius of curvature

**2. Find the radius of curvature at the point of the curve (VTU 2010)**

Solution: Let the given curve is

**Differentiating with respect to x, we get**

**(1)**

**Differentiate (1) with respect to x, we get**

**At the radius of curvature**

**3. Show that the radius of curvature at on the curve is**

**(VTU 2000S, 2014)**

Solution: Let the given curve is

**Differentiating with respect to x, we get**

**(1)**

**Differentiating (1) with respect to y, we get**

**At the radius of curvature**

**4. Show that the radius of curvature at of the curve is . (2009S)**

Solution: Let the given curve is

**Differentiating with respect to x, we get**

**(2)**

At **,**

**Differentiating (1) with respect to x, we get**

At **,**

**At the radius of curvature**

**5. For the curve show that . (VTU 2008)**

Solution: Let the given curve is

**Differentiate with respect to x, we get**

**(1)**

**[]**

**Differentiate (1) with respect to x, we get**

**[]**

**Therefore the radius of curvature**

Squaring on both sides we get

Raising the power 1/3 on both sides,

or

**6. Show that the radius of curvature for the curve where it crosses the line   
 is .**

Solution: Let the given curve is

**Differentiate with respect to x, we get**

**(1)**

**Differentiate (1) with respect to x,**

**Therefore the radius of curvature**

Solving the two equation and , we get

**and**

**At**

**8. Find the radius of curvature at any point of the parabola**

Solution: Let the given curve is

**Differentiate with respect to x, we get**

**(1)**

**At the point**

**Differentiate (1) with respect to x, we get**

**At the**

**Therefore the radius of curvature**

**9. Find the radius of curvature at the origin for**

**(i) (ii) .**

**(iii)**

**(i) Solution: Let the given curve is**

**Differentiate with respect to x, we get**

**(1)**

**At the point**

**Differentiate (1) with respect to x, we get**

**At the point**

**Therefore the radius of curvature**

**(ii) .**

**(i) Solution: Let the given curve is**

**Differentiate with respect to x, we get**

**(1)**

**At the point**

**Rewrite equation (1),**

**Differentiate with respect to y, we get**

**At the point**

**Therefore the radius of curvature**

**(iii)**

**Solution: Let the given curve is**

**Differentiate with respect to x, we get**

**(1)**

**At (0, 0)**

**Differentiate (1) with respect to x,**

**At (0, 0)**

**Therefore the radius of curvature**

**Problems: (Parametric form)**

**1. Show that the radius of curvature at any point of the cycloid**

**is .**

**Solution:** Let the given curve is

**Differentiate with respect to, we get**

**and**

**Differentiate again with respect to , we get**

**and**

**Consider,**

**And**

**Therefore the radius of curvature**

**2. Prove that the radius of curvature at any point of the Astriod is three   
 times the length of the perpendicular from the origin to the tangent at that point.**

**Similarly find radius of curvature of at (1, 1) (2017)**

**Solution:** Let the parametric equation of the given curve is

**and**

**Differentiate with respect to, we get**

**and**

**Differentiate again with respect to , we get**

**and**

**Consider,**

**And**

**Therefore the radius of curvature**

**3. Find the radius of curvature at any point on the ellipse:**

**Solution:** Let the given curve is

**Differentiate with respect to, we get**

**and**

**Differentiate again with respect to , we get**

**and**

**Consider,**

**And**

**Therefore the radius of curvature**

**4. Find the radius of curvature at any point on the curve**

**.**

**Solution:** Let the given curve is

**Differentiate with respect to t, we get**

**and**

**Differentiate again with respect to t, we get**

**and**

**Consider,**

**And**

**Therefore the radius of curvature**

**5. Show that the radius of curvature at the point t on the curve is**

**Solution:** Let the given curve is

**Differentiate with respect to t, we get**

**and**

**Differentiate again with respect to t, we get**

**and**

**Consider,**

**And**

**Therefore the radius of curvature**

**6. Show that the radius of curvature at each point of the curve   
 is inversely proportional to the length of the normal intercepted between the point   
 on the curve and the x-axis.**

**Solution:** Let the given curve is

**Differentiate with respect to t, we get**

**Differentiate again with respect to , we get**

**Now,**

**Differentiate again with respect to , we get**

**Consider,**

**And**

**Therefore the radius of curvature**

**3. Radius of curvature for polar curve is given by:**

**Proof:** With the usual notations, from the fig.

p

0

X

M

r

**Differentiating with respect to s,**

(1)

Also we know that

where

(2)

Also (3)

**Substituting the value from (2) and (3) in (1),**

or

**4. Radius of curvature for pedal curve is given by: .**

**Proof:** Radius of curvature for pedal curve

From the above figure with usual notation we have

**Differentiating with respect to s**,   
 (1)

Also we know that

[]

[from equation (1)]

**or**

**Polar and pedal form:**

**1. Show that the radius of curvature at any point of the cardioids**

**varies as . (VTU2003)**

**Solution:** Given a polar curve

**Differentiating with respect to**

**and**

**the radius of curvature is**

**(1)**

**The radius of curvature varies as .**

**Also squaring on both sides of the equation (1), we get**

**.**

**2. (2017)**

**Solution:** Given a polar curve

**Taking log on both sides we get**

**Differentiating with respect to**

**Differentiating with respect to**

**and given the polar curve**

**The radius of curvature is**

**The radius of curvature varies inversely as .**

**3. Find the radius of curvature for the parabola .**

**Solution:** Given a polar curve

**Taking log on both sides, we get**

**Differentiating with respect to**

**and**

**and (1)**

**The radius of curvature is**

**Hence**

**The radius of curvature varies as**

**4. If be the radii of curvature at the extremities of any chord of the cardoiod**

**which passes through the pole, show that**

**5. For any curve , Prove that .**

**6. Prove that for the ellipse in pedal form , the radius of curvature at the   
 point is**